Relations And Functions

Cartesian product : Given two non-empty sets P and Q. The cartesian Product $P \times Q$ is the set of all ordered pair of elements from P and Q i.e., $\boxed{P \times Q = \{(p,q) : p \in P,\ q \in Q\}}$

If either P or Q is the null set, then $P \times Q$ will also be empty set, i.e. $P \times Q = \phi$

If $A = \{a_1, a_2\}$ and $B = \{b_1, b_2, b_3, b_4\}$ then $A \times B = \{(a_1, b_1), (a_1, b_2), (a_1, a_3), (a_1, b_4), (a_2, b_1), (a_2, b_2), (a_2, b_3), (a_2, b_4)\}$

Note : (i) Two ordered pairs are equal, if and only if the corresponding first elements are equal and the second elements are also equal.

(ii) If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

(iii) If A and B are non-empty sets and either A or B is an infinite set then, $A \times B$ is also a infinite set.

(iv) $A \times A \times A = \{(a,b,c) : a, b, c \in A\}$. Here (a,b,c) is called an ordered triplet.

Relations :

A Relation R from a non-empty set A to a non-empty set B is a subset of the cartesian product $A \times B$. The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in $A \times B$. The second element is called the image of the first element.

Domain : The set of all first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R.

Range : The set of all second elements in a relation R from a Set A to Set B is called the range of the relation R.

Codomain : The whole set B is called the codomain of the relation R.

$\boxed{\text{range} \subseteq \text{codomain}}$

Note : A Relation R from A to A is also stated as a relation on A.

Note : If $n(A) = p$ and $n(B) = q$,

then, $n(A \times B) = pq$

the total no. of relation is $= 2^{pq}$

function : A relation f from a Set A to a Set B is said to be a function if every element of Set A has one and only one image in Set B.

If f is a function from A to B and $(a,b) \in f$, then b is called the image of a under f and a is called the preimage of b under f. The function f from A to B is denoted by $f : A \to B$

Real valued function : A function which has either R or one of its subsets as its range is called a real valued function

Some functions :

1. **Identity function** : Let R be the set of real numbers. Define the real valued function $f : R \to R$ by $y = f(x) = x$ for each $x \in R$. Such a function is called the identity function.

2. **Constant function** : Define the function $f : R \to R$ by $y = f(x) = c$, $x \in R$ where c is a constant and each $x \in R$. Here domain of f is R and its range is $\{c\}$.

3. **Polynomial function** : A function $f : R \to R$ is said to be polynomial function if for each x in R, $y = f(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$, where n is a non-negative integer and $a_0, a_1, a_2, \dots, a_n \in R$.

4. **Rotational function** : Rotational functions are function of the type $\frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are polynomial functions of x defined in a domain, where $g(x) \neq 0$.

5. **The modulus function** : The function $f : R \to R$ defined by $f(x) = |x|$ for each $x \in R$ is called modulus function

6. **Signum function** : The function $f : R \to R$ defined by $f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$ is called the signum function

7. **Greatest integer function** : The function $f : R \to R$ defined by $f(x) = [x]$, $x \in R$ assumes the value of the greatest integer, less than or equal to x. Such a function is called the greatest integer function.

Algebra of real functions : Let $f: X \to R$ & $g: X \to R$

1. **Addition of two real functions** : $\boxed{(f+g)(x) = f(x) + g(x)}$ for all $x \in X$

2. **Subtraction of a real function from another** : $\boxed{(f-g)(x) = f(x) - g(x)}$ for all $x \in X$

3. **Multiplication by a scalar** : $\boxed{(\alpha f)(x) = \alpha f(x)}$, $x \in X$

4. **Multiplication of two real functions** : $\boxed{(fg)(x) = f(x)\, g(x)}$ for all $x \in X$ (pointwise multiplication)

5. **Quotient of two real functions** : $\boxed{\left[\frac{f}{g}\right](x) = \frac{f(x)}{g(x)}}$, provided $g(x) \neq 0$, $x \in X$.